

APPLICATION OF THE SIMILARITY THEORY TO THE STUDY  
OF OPTOTHERMODYNAMIC PROCESSES IN LIQUIDS

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The article suggests a method of calculating the pressure in an absorbing liquid exposed to radiation.

The effect of powerful radiant fluxes on a substance, as a rule, is that the thermodynamic state of the substance undergoes a substantial change. This effect is often the simplest or even the only possible way of converting a thermodynamic system from the initial state on a specified path into the required final state (deep intrusion into the range of metastability, impulsive transition in the vicinity of the critical point, etc.). This circumstance stimulated the intensive investigation of various optothermodynamic phenomena that has been conducted recently both theoretically and experimentally (see [1] and the literature cited there).

However, these phenomena depend strongly on the individual properties of the tested substances, and they may change substantially upon transition from one object of investigation to another. Therefore, to make the results more general, it is natural to use methods of the similarity theory so as to obtain some universal relationships. Such an approach was also used in our work. We examined the problem of pressure arising in an absorbing liquid under the effect of radiation. The liquid occupies a sealed cuvette with volume  $V$  which is exposed to radiation through the lateral surface with area  $S$  by a uniform radiant flux of intensity  $q(t)$ .

Apart from being of interest in itself, such a problem is also of great applied importance because it stimulates processes occurring in cooling systems of solid-state lasers [2, 3].

We assume that the characteristic time of action of the radiant pulse  $\tau$  is considerably shorter than the time of equalizing the temperature profile forming in the liquid:  $K^2\chi\tau \ll 1$ , but considerably longer than the time of equalizing the pressure arising in the entire bulk of the liquid:  $c_0\tau \gg \delta$ , where  $c_0$  is the adiabatic speed of sound in the liquid under the initial conditions. In addition to that, we will assume mass absorption of the radiation so that  $K = \mu\rho$ ,  $\mu = \text{const}$ , and that the optical thickness of the cuvette is great:  $\mu\rho_0\delta \gg 1$ . These constraints are fulfilled, as a rule, in all cases that are of practical interest [2, 3].

With the above assumptions the pressure arising in the liquid may in the general case be represented as a function of the following parameters:

$$p = p(\mu, \rho_0, p_0, p_*, \omega_0, \omega_*, \delta, I). \quad (1)$$

As the set of thermodynamic variables we found in convenient not to use density and temperature, but density and specific enthalpy ( $\rho, w$ ). In writing relation (1) we took into account that the liquid obeys the theorem of corresponding states

$$\rho/\rho_* = F(\pi, j), \quad (2)$$

where  $\pi = p/p_*$ ,  $j = w/w_*$ , and  $w$  is counted from absolute zero. That is why  $\rho_*$  is not contained in (1): it can be expressed through  $\rho_0$  and  $F(1,1)$  from (2).

In the problem under examination there are three independent dimensionalities:  $m, kg, sec$ ; therefore, according to the  $\pi$ -theorem [4], out of the eight parameters contained in (1) we can form five independent dimensionless complexes, which it is convenient to choose in the following form:

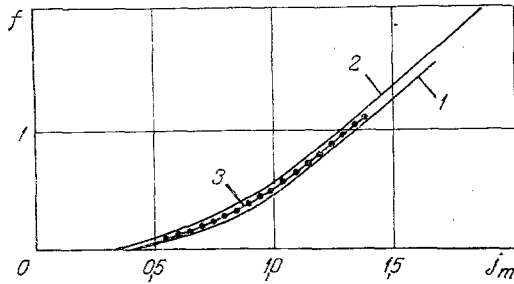


Fig. 1

Fig. 1. Dependence of the function  $f(j_0, j_m)$  on the reduced specific enthalpy  $j_m$ : 1) water; 2) benzene; 3) alcohol.

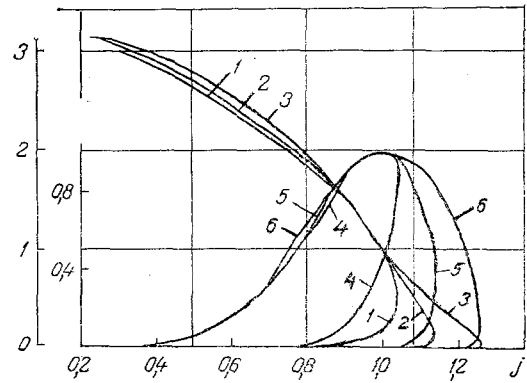


Fig. 2

Fig. 2. Reduced density  $\tilde{\rho}$  on the isobar for critical pressure (1-3) and reduced pressure  $\pi$  on the binodal curve (4-6) vs reduced specific enthalpy of liquids: 1, 4) benzene; 2, 5) alcohol; 3, 6) water.

$$j_0 = w_0/w_*; \pi = p_0/p_*; \Lambda_0 = K\delta; j_m = \mu l/w_*; \Xi = p_*/\rho_0 c_0^2.$$

The adiabatic speed of sound  $c_0$  is determined from the equation of state (2). Then (1) has the form

$$p = \rho_0 c_0^2 \pi(\Lambda_0, \Xi, \pi_0, j_0, j_m), \quad (3)$$

where  $\pi$  is a universal dimensionless function for all substances.

Expression (3) is the strict result following from the  $\pi$ -theorem. It follows from (3) that in two different cooling systems  $p/\rho_0 c_0^2$  is the same if and only if in these systems the following five dimensionless complexes are equal:  $\Lambda_0, \Xi, \pi_0, j_0$ , and  $j_m$ . Such a large number of similarity criteria makes (3) unsuitable for practical use. However, by somewhat reducing the accuracy, the number of these criteria may be reduced, too.

We will make use of the fact that usually  $\pi_0 \ll 1, \Xi \ll 1, \Lambda_0 \gg 1$ , and we expand the right-hand side of (3) into a series in terms of  $\Lambda_0^{-1}, \pi_0$ , and  $\Xi$ , confining ourselves to the first nonzero terms.

We note that with fixed values of the other parameters, the condition  $\Lambda_0 \rightarrow \infty$  may be satisfied if  $\delta \rightarrow \infty$ , but with  $\delta \rightarrow \infty, p \rightarrow p_0$  because the final energy absorbed by the liquid is distributed in that case over an infinite volume. Then the zero term of the expansion in terms of  $\Lambda_0^{-1}$  is equal to  $p_0$ , which is obvious, so that we must take into account the term of first order in terms of  $\Lambda_0^{-1}$ . In terms of  $\pi_0$  and  $\Xi$  we may confine ourselves to the zeroth approximation. Taking this into account, we may write (3) in the form

$$p - p_0 = \frac{\rho_0 c_0^2}{\Lambda_0} f(j_0, j_m), \quad (4)$$

where  $f(j_0, j_m)$ , in accordance with [2], has the following form in the reduced variables:

$$f(j_0, j_m) = \int_{j_0}^{j_m} \frac{[\tilde{\rho}_0 - \tilde{\rho}(j)]}{\int_0^j \tilde{\rho}(j') dj'} dj. \quad (5)$$

Figure 1 shows the shape of the function  $f(j_0, j_m)$  with  $j_0 = 0.33$  for three liquids: water, benzene, and ethyl alcohol (azeotropic mixture with water), whose thermodynamic parameters (5) have been most fully tabulated. The value  $j_0 = 0.33$  corresponds to 6°C for benzene, to 20°C for alcohol, and to 60°C for water. These liquids are often used as heat carriers in cooling systems.

It follows from Fig. 1 that the function  $f(j_0, j_m)$  is sufficiently universal, regardless of the fact that two of the liquids (water and alcohol) belong to the associated liquids which badly satisfy the theorem of corresponding states.

TABLE 1. Values of the Function  $f(j_0, j_m)$  Calculated for Alcohol

$i_0$	$i_m$				
	0,5	0,75	$\sqrt{\quad}$	1,25	1,5
0,2	0,2	0,4	0,6	0,95	1,3
0,3	0,1	0,3	0,6	0,95	1,35
0,4	—	0,3	0,6	1,0	1,45
0,5	—	0,2	0,5	1,0	1,5

Note. Intermediate values of the function can be determined by linear interpolation.

The obtained expression (4) makes it possible to determine the pressure in the absorbing liquid according to the values of  $j_0$ ,  $j_m$ , and  $\rho_0 c_0^2$ , and also the form of the function  $f(j_0, j_m)$ , whose values are given in Table 1. Since  $w_*$ , counted from absolute zero and indispensable for calculating  $j_0$  and  $j_m$ , does not appear in the literature, we found it by using the dependence of heat capacity on temperature in the range  $t^\circ < 0^\circ\text{C}$  and the tabulated values of critical enthalpy counted from  $0^\circ\text{C}$ . The respective values of  $w_*$  for water, benzene, and alcohol are 2.725, 1.06, and 1.52 MJ/kg.

It should be pointed out that the theorem of corresponding states for vapors is applicable only in a narrow interval near the critical point; this is illustrated by the graphs of the dependence of  $\beta = \rho/\rho_*$  and  $\pi = p/p_*$  on  $j$  presented in Fig. 2. Our analysis is therefore correct only in the range of parameters corresponding to the absence of boiling of the liquid.

The suggested method of finding the pressure in a liquid may be used in practical calculations of cooling systems where there is no boiling.

In conclusion, we note that relation (4), obtained in [2] by complicated calculations, was derived by us in the present article from very simple considerations based on using the  $\pi$ -theorem. It follows that the use of methods of the theory of similarity in problems of optothermodynamics may prove to be an efficient method of investigating various phenomena.

#### NOTATION

$V$ , volume of the cuvette with the liquid;  $S$ , area of the irradiated surface;  $q$ , intensity of the incident radiation;  $t$ , time;  $\tau$ , time of radiation pulse acting;  $K$ , absorption coefficient;  $\chi$ , thermal diffusivity of the liquid;  $c_0$ , adiabatic speed of sound in the liquid;  $\delta = V/S$ , characteristic dimension of the cuvette in the direction of propagation of the radiation;  $\rho$ , density of the liquid;  $\mu$ , absorption coefficient per unit density;  $p$ , pressure of the liquid;  $w$ , enthalpy;  $I = \int_0^t q(t') dt'$ , spent energy at the instant  $t$ ;  $\pi = p/p_*$ , dimensionless pressure;  $j = w/w_*$ , dimensionless enthalpy;  $\Lambda = K\delta$ , dimensionless optical thickness of the cuvette;  $j_m = \mu I/w_*$ , dimensionless spent energy;  $\Xi = \rho_0 c_0^2$ , dimensionless critical pressure. Subscripts: 0 and \*, initial and critical state, respectively.

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